

Mechanism of Generation of Black Hole Entropy in Sakharov's Induced Gravity

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Abstract

The mechanism of generation of the Bekenstein-Hawking entropy S^{BH} of a black hole in the Sakharov's induced gravity is proposed. It is suggested that the "physical" degrees of freedom, which explain the entropy S^{BH} , form only a finite subset of the standard Rindler-like modes defined outside the black hole horizon. The entropy S_R of the Rindler modes, or entanglement entropy, is always ultraviolet divergent, while the entropy of the "physical" modes is finite and it coincides in the induced gravity with S^{BH} . The two entropies S^{BH} and S_R differ by a surface integral Q interpreted as a Noether charge of non-minimally coupled scalar constituents of the model. We demonstrate that energy E and Hamiltonian H of the fields localized in a part of space-time, restricted by the Killing horizon Σ , differ by the quantity $T_H Q$, where T_H is the temperature of a black hole. The first law of the black hole thermodynamics enables one to relate the probability distribution of fluctuations of the black hole mass, caused by the quantum fluctuations of the fields, to the probability distribution of "physical" modes over energy E . The latter turns out to be different from the distribution of the Rindler modes. We show that the probability distribution of the "physical" degrees of freedom has a sharp peak at $E = 0$ with the width proportional to the Planck mass. The logarithm of number of "physical" states at the peak coincides exactly with the black hole entropy S^{BH} . It enables us to argue that the energy distribution of the "physical" modes and distribution of the black hole mass are equivalent in the induced gravity. Finally it is shown that the

Noether charge Q is related to the entropy of the low frequency modes propagating in the vicinity of the bifurcation surface Σ of the horizon. We find in particular an explicit representation of Q in terms of an effective action of some two-dimensional quantum fields "living" on Σ .

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1 Introduction

Searching for statistical-mechanical explanation of the Bekenstein-Hawking [1]–[3] entropy S^{BH} of black holes attracted a lot of attention in the last years. In particular, one of the proposed ideas was to relate S^{BH} to counting of quantum excitations of a black hole [4]–[12]. This suggestion, however, meets a difficulty because the Bekenstein-Hawking entropy arises at tree-level while the entropy of quantum excitations is a one-loop quantity. As was first pointed out in [13], this difficulty can be resolved in the Sakharov’s theory of induced gravity [14],[15]. According to the Sakharov’s idea general relativity can be considered as a low energy effective theory where the metric $g_{\mu\nu}$ becomes dynamical variable as the result of quantum effects in the system of heavy constituents propagating in the external gravitational background. Gravitons in this picture up to some extend are analogous to the phonon field describing collective excitations of a lattice in low-temperature limit of the theory.

Surely, the Sakharov’s approach does not provide us with a complete understanding of gravity at Planckian scales and it cannot compete, for instance, with superstring models. Nevertheless, it has a number of features, such as description of the graviton as a collective variable and absence of the leading one-loop divergencies, which, in accordance with our intuition, should be the key properties of more profound candidates to the role of quantum gravity theory. One may hope, in particular, that studying the Bekenstein-Hawking entropy in the framework of the induced gravity would give us some hints how the entropy can be explained in more realistic models and what is the origin of its universality.

Recently we proposed a model [16] which illustrates explicitly how the black hole entropy S^{BH} is generated in Sakharov’s induced gravity. Namely, it was shown that S^{BH} is directly related to the statistical-mechanical entropy S_R of the thermally excited gas of the heavy constituents (Rindler-like particles) propagating in the close vicinity of the black hole horizon

$$S^{BH} = S_R - \bar{Q} \quad . \quad (1.1)$$

Both fermions and bosons give positive and infinite contributions to S_R , so that this quantity is divergent. An additional term \bar{Q} in (1.1) is proportional to the fluctuations of the non-minimally coupled scalar fields $\hat{\phi}_s$ on the horizon Σ and is the average value of the following operator

$$\hat{Q} = 2\pi \sum_s \xi_s \int_{\Sigma} \hat{\phi}_s^2 \sqrt{\gamma} d^2x \quad , \quad (1.2)$$

where ξ_s are the corresponding non-minimal couplings. The presence of such couplings is an important property of the model that allows one to induce the gravitational action with the finite Newton constant G . The remarkable property of the model is that for the same values of the parameters of the constituents, that guarantee the finiteness of G , the divergences of S_R are exactly cancelled by the divergences of \bar{Q} . So in the induced gravity the right-hand-side (r.h.s.) of (1.1) is always finite and reproduces exactly the

Bekenstein-Hawking expression $S^{BH} = \mathcal{A}^H/(4G)$, where \mathcal{A}^H is the surface area of the black hole.

Let us note that the operator \hat{Q} has a clear interpretation as a Noether charge. In Wald's approach [17] \hat{Q} is that part of the Noether charge which arises because of non-minimal couplings of the scalar fields $\hat{\phi}_s$ [18]. This fact, being virtually unimportant for classical black holes which cannot have scalar hair [19], becomes crucial in the quantum theory where fields have non-zero fluctuations $\langle \hat{\phi}_s^2 \rangle$. The Noether charge interpretation also gives us a hint how the generalization of formula (1.1) for the black hole entropy might look in more realistic models.

Eq.(1.1) shows that the entropy S_R of the thermal bath outside the black hole is much larger than the quantity S^{BH} and thus it overcounts the degrees of freedom required to reproduce the Bekenstein-Hawking entropy. Also, as has been pointed out by several authors [20]-[23], on Ricci-flat geometries the entropy S_R ignores non-minimal couplings¹, while massive fields contribution to S^{BH} depends on constants ξ_s .

In this paper we analyse the following two questions. First, what is the mechanism which enables one to separate the "physical" modes responsible for the entropy S^{BH} from other thermal excitations, and, second, what is the statistical-mechanical meaning of the quantity \bar{Q} in Eq.(1.1).

We begin with the observation that the Hamiltonian H of a non-minimally coupled scalar field calculated for the black hole exterior differs from energy E calculated with the help of the stress-energy tensor $T_{\mu\nu}$. The difference $T_H Q$ is proportional to the Noether charge Q , where T_H is the Hawking temperature of the black hole. The quantity E defines the difference between the mass of the black hole at the horizon and the mass of the system measured at infinity. Thus for the fixed mass at infinity fluctuations of quantum constituent fields result in the quantum fluctuations of the black hole mass.

On the other hand, the value of the Hamiltonian H coincides with the canonical energy. For a fixed temperature Rindler-like modes of constituents are thermally distributed with respect to the canonical energy. Hence the Hamiltonian H allows one to calculate the statistical-mechanical entropy S_R which enters Eq.(1.1). Formula (1.1) indicates that "physical" modes that are responsible for the black hole entropy S^{BH} form a subset of the total set of the Rindler modes. We shall show that the difference \bar{Q} between S_R and S^{BH} is directly connected with the difference between the energy E and the canonical energy H and is defined by the fluctuations of the non-minimally coupled fields at the horizon. It will be demonstrated that \bar{Q} is completely determined by zero-frequency ("soft") modes of scalar fields propagating in the vicinity of the horizon. We shall also

¹By imposing on the field near the horizon special boundary conditions depending on ξ one can make S_R depending on ξ as well. Moreover recently Solodukhin [24] suggested such a scattering condition on the horizon which enables one to reproduce the term \bar{Q} in the entropy (1.1). However the physical meaning of this scattering condition is not clear.

show that the statistical-mechanics of the soft modes is equivalent to a two-dimensional (2D) quantum theory of effective fields (fluctons) "living" on the bifurcation surface Σ .

We argue that the leading temperature asymptotics of the canonical ensembles of Rindler, "physical" and soft modes in the induced gravity have a universal form because they are determined only by the behavior of the system near the horizon. It enables us to find for Ricci-flat backgrounds the distribution of the "physical" degrees of freedom explicitly. We show, in particular, that at the given temperature the probability distribution of "physical" states has a sharp peak near the average energy $E = 0$ with the width determined by the masses of the heaviest constituents of the induced gravity models. The logarithm of number of physical states at the peak is exactly the Bekenstein-Hawking entropy.

Thus the proposed mechanism of the entropy generation in Sakharov's induced gravity implies that: i) the space of Rindler modes consists of subspaces of "physical" and soft modes, and ii) the density number of "physical" states at $E = 0$ determines the degeneracy of the black hole mass spectrum. The obtained results confirm the consistency of these suggestions.

The paper is organized as follows. In Section 2 we formulate the model of induced gravity and recall some results of Ref.[16] that are necessary for our consideration. In Section 3 we discuss a non-minimally coupled scalar field defined on a part of black hole background restricted by the Killing horizon and find out the relation between its energy and Hamiltonian. In Section 4 we show that the fluctuations of a scalar field on the bifurcation surface Σ of the Killing horizons is determined only by the contribution of the soft modes. We also obtain the canonical thermal average of $\langle \hat{\phi}^2 \rangle_\beta$ for the Rindler space. The relation between the degeneracy of the black hole mass spectrum and spectral density of "physical" modes is discussed in Section 5. In Section 6 we calculate the spectral density of Rindler states and discuss its properties. Spectral densities of the soft and "physical" modes are found in Section 7. These results are used to obtain the probability distribution of the black hole mass and degeneracy of the black hole spectrum. In Section 8 and Appendix we demonstrate that the statistical mechanics of soft modes can be related to degrees of freedom of an effective 2D theory. Namely, we show that the charge \bar{Q} is expressed in terms of a 2D effective action of some massive quantum scalar fields "living" on the surface Σ . Section 9 contains discussion of the results.

We use sign conventions of the book [25], and thus use the signature $(-, +, +, +)$ for a Lorentzian metric.

2 Induced entropy of a black hole in Sakharov's induced gravity

We recall that the starting point of the induced gravity approach is the equality

$$\exp(-W[g_{\mu\nu}]) = \int \mathcal{D}\Phi_i \exp(-I[\Phi_i, g_{\mu\nu}]) \quad , \quad (2.1)$$

that expresses the effective action $W[g_{\mu\nu}]$ of the gravitational low energy effective theory in terms of a quantum average of the constituent fields Φ_i propagating in a given external gravitational background $g_{\mu\nu}$. The Sakharov's basic assumption is that the gravity becomes dynamical only as the result of quantum (one-loop) effects of the constituent fields.

A simple model convenient for the discussion of the problem of black hole entropy in induced gravity was suggested in [16]². This model is built of N_s free scalar bosons ϕ_s with masses m_s and of N_d free fermion fields ψ_d with masses m_d . The scalar fields have non-minimal couplings with constants ξ_s and their classical actions $I[\phi_s, g_{\mu\nu}]$ are similar to the scalar action which will be considered in Section 3, see Eq.(3.1). The fermion fields are the Dirac spinors ψ_d with the Dirac actions $I[\psi_d, g_{\mu\nu}]$. Thus effective gravitational action $W[g_{\mu\nu}]$ is defined by Eq. (2.12) where the classical action for the constituent fields ($\Phi_i = \{\phi_s, \psi_d\}$) is the sum

$$I[\Phi_i, g_{\mu\nu}] = \sum_s I[\phi_s, g_{\mu\nu}] + \sum_d I[\psi_d, g_{\mu\nu}] \quad . \quad (2.2)$$

Consider now the following two functions

$$p(z) = \sum_s m_s^{2z} - 4 \sum_d m_d^{2z} \quad , \quad q(z) = \sum_s m_s^{2z} (1 - 6\xi_s) + 2 \sum_d m_d^{2z} \quad (2.3)$$

constructed from the parameters of the constituents. Direct calculations show that the induced cosmological constant vanishes and the induced gravitational coupling constant G is finite if the following constraints are satisfied

$$p(0) = p(1) = p(2) = p'(2) = 0 \quad , \quad (2.4)$$

$$q(0) = q(1) = 0 \quad . \quad (2.5)$$

In particular, the condition $p(0) = 0$ requires that $N_s = 4N_d$ and is always satisfied in supersymmetric theories. The finite Newton constant G is the function of the parameters of the constituents

$$\frac{1}{G} = \frac{1}{12\pi} q'(1) = \frac{1}{12\pi} \left(\sum_s (1 - 6\xi_s) m_s^2 \ln m_s^2 + 2 \sum_d m_d^2 \ln m_d^2 \right) \quad . \quad (2.6)$$

²Another discussion of black hole entropy in induced gravity can be found in [26].

For $N_d > 1$ and $N_s > 4$ the equations (2.4) for masses of the constituents are consistent, while equations (2.5) are linear equations defining ξ_s . Relation (2.6) shows that some of the fields (heavy constituents) have masses comparable to the Planck mass.

The heavy constituents are unobservable at low energies and their effective action $W[g_{\mu\nu}]$ is reduced in the low energy regime to the Einstein-Hilbert action

$$W[g_{\mu\nu}] = -\frac{1}{16\pi G} \left(\int_{\mathcal{M}} dV R + 2 \int_{\partial\mathcal{M}} dv K \right) + \dots \quad (2.7)$$

The dots in r.h.s. of (2.7) indicate higher curvature terms which are suppressed by the power factors of m_i^{-2} when the curvature is small. In order to make finite the terms which are quadratic in curvature one must consider a more general set of constituent fields with additional constraints imposed on them. For our problem, since we will be interested in Ricci-flat geometries, possible local \mathcal{R}^2 -terms give a pure topological contribution to the action which is irrelevant for our discussion. For this reason in what follows we omit such terms.

The variation of $W[g_{\mu\nu}]$ gives the Einstein equations

$$\frac{\delta W}{\delta g_{\mu\nu}} \sim R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 0. \quad (2.8)$$

According to the Sakharov's equality (2.1) these equations in the induced gravity are equivalent to the relation

$$\langle \hat{T}^{\mu\nu}(x) \rangle = 0, \quad (2.9)$$

where $\hat{T}^{\mu\nu}(x)$ is the total stress-energy tensor of the constituents.

The value of the Einstein-Hilbert (2.7) action calculated on the Gibbons-Hawking instanton determines the classical free energy of the black hole, and hence gives the Bekenstein-Hawking entropy. The remarkable feature of the Sakharov's equality (2.1) is that it allows one to rewrite identically the same classical free energy of the black hole in terms of average over the heavy constituents in the Hartle-Hawking state propagating on the black hole background. By using this representation it is possible to get an explicit expression for the Bekenstein-Hawking entropy in terms of constituents [16]

$$S^{BH} = - \sum_i \text{Tr } \hat{\rho}_i \ln \hat{\rho}_i - \sum_s 2\pi \xi_s \int_{\Sigma} \langle \hat{\phi}_s^2 \rangle \sqrt{\gamma} d^2x \quad (2.10)$$

The thermal density matrix $\hat{\rho}_i$ of the Rindler particles for constituents in the Hartle-Hawking state is

$$\hat{\rho}_i = \frac{e^{-\beta_H \hat{H}_i}}{\text{Tr } e^{-\beta_H \hat{H}_i}} \quad (2.11)$$

Here \hat{H}_i are Hamiltonians of the fields and β_H is the inverse Hawking temperature. Equation (2.10) shows that S^{BH} is related to the statistical-mechanical entropy $S_R = - \sum_i \text{Tr } \hat{\rho}_i \ln \hat{\rho}_i$ of the heavy constituents computed on the given black hole background³,

³The entropy S_R can be also interpreted as an entanglement entropy, see [5],[6],[10].

and also depends on the average of the square of the scalar field operators on the black hole horizon Σ .

Note that each term $-\text{Tr } \hat{\rho}_i \ln \hat{\rho}_i$ in (2.10) is positive and divergent. The last term in the r.h.s. of (2.10) appears because of non-minimal couplings. Presence of such couplings in the scalar sector of the model is imperative in order to provide the ultraviolet finiteness of the Newton constant in the low-energy gravitational action. What is remarkable, the terms with non-minimal couplings exactly cancel all the divergencies in S_R so that r.h.s of (2.10) correctly reproduces the Bekenstein-Hawking entropy.

Strictly speaking the Bekenstein-Hawking entropy is only the leading part of the r.h.s. of (2.10). Since the curvature \mathcal{R} of the spacetime does not vanish relation (2.10) also contains the corrections of the order $m_i^{-2}\mathcal{R}$. These terms are directly related to the higher order in curvature corrections to the Einstein-Hilbert action (2.7). The masses of the constituents are very high and their modes are thermally excited only in the narrow region in the vicinity of the horizon. For the description of these modes we shall use the Rindler approximation and omit the curvature dependent corrections to $-\text{Tr } \hat{\rho}_i \ln \hat{\rho}_i$ and $\langle \hat{\phi}_s^2 \rangle$ that are of the same order $m_i^{-2}\mathcal{R}$ as the terms omitted in (2.7).

After these remarks let us discuss the concrete mechanism of cancellation of divergences in (2.10). The partition function for a massive field with the mass m_i can be calculated explicitly in the limit when the curvature radius of the space-time is much larger then the Compton wave length of the field

$$\text{Tr } e^{-\beta \hat{H}_i} \simeq \exp(-\mu_i \beta - U_i(\beta)) \quad (2.12)$$

(see for the details Ref.[16]). Here β^{-1} is the temperature of the system measured at infinity, \hat{H}_i is the Hamilton operator of the field in question, μ_i is a parameter associated to the vacuum energy and

$$U_i(\beta) = -g(m_i^2) \frac{\pi}{6} \frac{\beta_H}{\beta} \mathcal{A}^H \quad . \quad (2.13)$$

Here $\mathcal{A}^H = \int_{\Sigma} \sqrt{\gamma} d^2 x$ is the area of the horizon. The function $g(m_i^2)$ depends on the mass m_i of the field and it is given by the integral

$$g(m_i^2) = n_i \int_0^\infty \frac{ds}{(4\pi s)^{D/2}} e^{-m_i^2 s} \quad , \quad (2.14)$$

where D is the dimensionality of the space time and the factor n_i is equal to 1 or 2 for scalars and (4D Dirac) fermions, respectively. This integral is divergent and it has to be regularized by using, for instance, the Pauli-Villars [21] or the dimensional [16] regularizations. The function $g(m_i^2)$ is important because it determines the statistical-mechanical entropy $-\text{Tr } \hat{\rho}_i \ln \hat{\rho}_i$ of the given field evaluated at $\beta = \beta_H$

$$-\text{Tr } \hat{\rho}_i \ln \hat{\rho}_i = \left(1 - \beta \frac{\partial}{\partial \beta}\right) \ln \text{Tr } e^{-\beta \hat{H}_i} = g(m_i^2) \frac{\pi}{3} \mathcal{A}^H \quad . \quad (2.15)$$

On the other hand, the same function $g(m_i^2)$ determines the average value of the scalar field $\langle \hat{\phi}_s^2 \rangle$ on the horizon

$$\int_{\Sigma} \langle \hat{\phi}_s^2 \rangle \sqrt{\gamma} d^2x = g(m_s^2) \mathcal{A}^H \quad . \quad (2.16)$$

Now, if entropies $-\text{Tr } \hat{\rho}_i \ln \hat{\rho}_i$ and averages (2.16) are regularized according to the same scheme with the equal regularization parameters, the substitution of Eqs.(2.15) and (2.16) into Eq.(2.10) gives the Bekenstein-Hawking entropy in the induced gravity

$$S^{BH} = \frac{1}{4G} \mathcal{A}^H \quad , \quad (2.17)$$

$$\frac{1}{G} = \frac{4\pi}{3} \left(\sum_s g(m_s^2)(1 - 6\xi_s) + \sum_d g(m_d^2) \right) \quad . \quad (2.18)$$

The Newton constant defined by (2.18) is ultraviolet finite, provided the constraints (2.4) and (2.5) are satisfied, and after regularization is removed its expression is given by Eq.(2.6). Formulas (2.17), (2.18) explicitly demonstrate the crucial role of the non-minimal coupling. It shows, in particular, that the total contribution of quantum fields to the black hole entropy can be finite only if $\xi_s > 0$ for some constituents.

3 Energy and Hamiltonian

We discuss now how the non-minimal coupling of scalar constituents manifests itself in the black hole thermodynamics. For simplicity in what follows we consider a static black hole. The consideration can be easily extended to the stationary case as well.

Let us recall that the complete background spacetime of an eternal black hole contains two Rindler-like wedges bounded by the Killing horizons. The Killing horizons intersect at the two dimensional bifurcation surface Σ . We shall use a foliation of the hypersurfaces $t=\text{const}$ orthogonal to the Killing vector ζ^μ that intersect each other at Σ .

Quantum fields propagate on the complete space-time manifold, however in our statistical-mechanical calculations we restrict ourselves by considering only a part of the system located in one of the wedges. It is instructive first to discuss how this procedure manifests in the classical theory. We focus on a classical scalar field ϕ with a non-minimal coupling with the scalar curvature R described by the action

$$I[\phi] = -\frac{1}{2} \int (\phi^{;\mu} \phi_{;\mu} + m^2 \phi^2 + \xi R \phi^2) \sqrt{-g} d^4x \quad . \quad (3.1)$$

The field obeys the equation

$$\square \phi - (m^2 + \xi R) \phi = 0 \quad , \quad (3.2)$$

where \square is the D'Alembert operator. The stress-energy tensor resulting from the variation of the action (3.1) with respect to the metric is

$$T_{\mu\nu} = \phi_{;\mu} \phi_{;\nu} - \frac{1}{2} g_{\mu\nu} (\phi_{;\rho} \phi^{;\rho} + m^2 \phi^2) + \xi \left[(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) \phi^2 + g_{\mu\nu} (\phi^2)_{;\rho}^{;\rho} - (\phi^2)_{;\mu\nu} \right] \quad . \quad (3.3)$$

Denote by \mathcal{B} a space-like hypersurface orthogonal to the Killing vector ζ^μ . The energy E of the system is defined in terms of the stress-energy tensor (3.3)

$$E = \int_{\mathcal{B}} T_{\mu\nu} \zeta^\mu d\sigma^\nu = - \int_{\mathcal{B}} T_0^0 \sqrt{-g} d^3x \quad , \quad (3.4)$$

where $d\sigma^\nu$ is the future directed vector of the volume element on \mathcal{B} . In the general case E differs from the *canonical* energy H that coincides with the Hamiltonian. The latter is expressed in terms of the Hamiltonian density \mathcal{H}

$$H = \int_{\mathcal{B}} \mathcal{H} \sqrt{-g} d^3x \quad , \quad (3.5)$$

where

$$\mathcal{H} = \frac{1}{2} \left(-g^{00} \phi_{,0}^2 + g^{ij} \phi_{,i} \phi_{,j} + (m^2 + \xi R) \phi^2 \right) \quad . \quad (3.6)$$

To compare E and H we note that in a static space-time

$$-T_0^0 = \mathcal{H} - \xi \left(R_0^0 \phi^2 + g^{ij} (\phi^2)_{;ij} \right) \quad . \quad (3.7)$$

The last term in r.h.s. of this equation can be rewritten as

$$\begin{aligned} (\phi^2)_{;ij} &= \square \phi^2 - g^{00} (\phi^2)_{;00} = g^{00} \left((\phi^2)_{,0,0} - (\phi^2)_{;00} \right) + \frac{1}{\sqrt{-g}} \partial_i \left(\sqrt{-g} g^{ij} \partial_j \phi^2 \right) = \\ &= \frac{1}{\sqrt{-g}} \partial_i \left(\sqrt{-g} g^{ij} ((\phi^2)_{,j} - \phi^2 w_j) \right) + \nabla_\mu w^\mu \phi^2 \quad . \end{aligned} \quad (3.8)$$

Here $w^\mu = \frac{1}{2} \nabla^\mu \ln |g_{00}|$ is a time-independent acceleration of the Killing observer. It can be shown that $\nabla_\mu w^\mu = -R_0^0$, so that for static space-times relation (3.7) takes the form

$$-T_0^0 = \mathcal{H} - \xi \frac{1}{\sqrt{-g}} \partial_i \left(\sqrt{-g} g^{ij} ((\phi^2)_{,j} - \phi^2 w_j) \right) \quad . \quad (3.9)$$

Then substitution of (3.9) into (3.4) gives the required relation between the energy E and the canonical energy H

$$E = H - \xi \int_{\partial \mathcal{B}} ds^k |g_{00}|^{1/2} ((\phi^2)_{,k} - \phi^2 w_k) \quad . \quad (3.10)$$

Here ds^k is a three dimensional vector in \mathcal{B} normal to the boundary $\partial \mathcal{B}$ and directed outward with respect to \mathcal{B} . Thus two energies differ by a surface term given on the boundary $\partial \mathcal{B}$ of the hypersurface \mathcal{B} .

Obviously, when one considers a complete Cauchy surface the boundary term in (3.10) contains only a contribution from the spatial infinity, or from the external spatial boundaries if they are present. For a field falling off at infinity or obeying suitable conditions at the boundary one can get rid of the boundary term and make E and H be equal.

However, the situation is qualitatively different when we consider the theory only in one of the wedges. Then the integration region in E is restricted by the bifurcation surface Σ of the Killing horizon, where the field ϕ can take arbitrary finite values. By assuming

that contribution from the spatial infinity or external boundary is absent one can write for the field regular at the horizon the following relation

$$E = H - \beta_H^{-1} Q \quad , \quad (3.11)$$

where β_H^{-1} is the Hawking temperature determined by the surface gravity κ of the black hole as $\beta_H^{-1} = \kappa/(2\pi)$ and

$$Q = 2\pi\xi \int_{\Sigma} \phi^2 \sqrt{\gamma} d^2x \quad . \quad (3.12)$$

We see therefore that the energy E computed for a domain restricted by the horizon differs from the canonical energy H by the quantity proportional to Q .

It is reasonable to ask what is the relevance of quantities E and H from the point of view of black hole thermodynamics. To this aim one can consider the field ϕ on a black hole background and calculate its contribution to the entropy and energy of a black hole. A simple way to do this is to make use of Euclidean formulation of the theory on the black hole instanton with the arbitrary period β of the Euclidean time. The results obtained in such an off-shell approach coincide with the results of other methods [27]-[29]. If $\beta \neq \beta_H$ the background has a conical singularity and one can write the Euclidean action in the form⁴

$$I_E[\phi, g_{\mu\nu}, \beta] = \frac{1}{2} \int_{\mathcal{M}_\beta} \left(\phi^{\mu} \phi_{,\mu} + m^2 \phi^2 + \xi R \phi^2 \right) \sqrt{-g} d^4x + 2\pi\xi \left(1 - \frac{\beta}{\beta_H} \right) \int_{\Sigma} \phi^2 \sqrt{\gamma} d^2x \quad . \quad (3.13)$$

Since for a static field configuration the bulk part of Euclidean action (3.13) is proportional to Hamiltonian (3.6) one can rewrite Eq.(3.13) as

$$I_E[\phi, g_{\mu\nu}, \beta] = \beta H + 2\pi\xi \left(1 - \frac{\beta}{\beta_H} \right) \int_{\Sigma} \phi^2 \sqrt{\gamma} d^2x \quad . \quad (3.14)$$

To derive the contributions ΔE and ΔS of the scalar field to the mass and entropy of the black hole one just identifies β^{-1} with the temperature and functional $\beta^{-1} I_E(\beta)$ with the free energy. If the scalar field does not vanish at the horizon one has

$$\Delta E = \frac{\partial}{\partial \beta} I_E[\phi, g_{\mu\nu}, \beta]|_{\beta=\beta_H} = H - \beta_H^{-1} Q \quad , \quad (3.15)$$

$$\Delta S = \left(\beta \frac{\partial}{\partial \beta} - 1 \right) I_E[\phi, g_{\mu\nu}, \beta]|_{\beta=\beta_H} = -Q \quad , \quad (3.16)$$

where Q is defined by Eq.(3.12). Thus the energy ΔE appears as a part of total energy of the system. As one can see from (3.15) and (3.16) the quantity Q contributes both to the mass and entropy of the black hole. Yet in the classical theory such contributions are absent when the field and metric obey the classical equations. This conclusion follows

⁴ As earlier we omit the terms connected with the external boundary that might be present. They are not important for our consideration and, as was explained, they can be avoided by choosing the appropriate boundary conditions.

from the recent analysis by Mayo and Bekenstein [19] who showed that for any value of the non-minimal coupling ξ a stationary black hole has no massive scalar hair.

The situation is quite different for a quantum field. Due to the presence of vacuum zero-point fluctuations the average $\langle \hat{\phi}^2 \rangle$ does not vanish. That is why quantum fluctuations of scalar fields on Σ manifest themselves in the black hole thermodynamics. Moreover, the contribution of a quantum scalar field to black hole entropy (2.10) because of non-minimal coupling directly follows from (3.16) if one replaces the classical quantity Q by its quantum version $\bar{Q} \equiv \langle \hat{Q} \rangle$ and takes the sum over all non-minimally coupled constituents.

4 Soft modes

Our aim now is to investigate the properties of the quantity Q . Because Q is defined strictly on Σ it is sufficient to consider only the behavior of scalar fields in the domain close to the horizon surface where the black hole metric can be approximated by the Rindler metric

$$ds^2 = -\kappa^2 \rho^2 dt^2 + d\rho^2 + (dz^1)^2 + (dz^2)^2 \quad , \quad (4.1)$$

where $\kappa = 2\pi/\beta_H$. Our aim is to demonstrate that the value of Q is determined only by a contribution of Rindler modes with negligibly small frequencies.

We begin with the analysis of Q in classical theory. A normalized solution of the classical Klein-Gordon equation in the Rindler space is [30]

$$U_{\omega,k}(x) = \frac{1}{2\pi} u_{\omega,k}(t, \rho) e^{-ik_j z^j} \quad , \quad (4.2)$$

$$u_{\omega,k}(t, \rho) = \frac{1}{2\pi^2} (\sinh \pi\omega)^{1/2} K_{i\omega}(\mu\rho) e^{-i\kappa\omega t} \quad , \quad (4.3)$$

where ω is the dimensionless frequency, $\mu = (m^2 + k_j^2)^{1/2}$, $j = 1, 2$, and $K_{i\omega}(x)$ is the modified (hermitean) Bessel function which vanish at $\rho \rightarrow \infty$. An interesting observation concerning these modes is that only modes with negligibly small frequencies ω contribute to the value of the field ϕ on the Rindler horizon. We call such solutions *soft* modes. Their behavior near the horizon follows from the asymptotic of the modified Bessel functions at small values of $\mu\rho$

$$K_{i\omega}(\mu\rho) \simeq \frac{i\pi}{2 \sinh \pi\omega} \left[\frac{1}{\Gamma(i\omega + 1)} \left(\frac{\mu\rho}{2} \right)^{i\omega} - \frac{1}{\Gamma(-i\omega + 1)} \left(\frac{\mu\rho}{2} \right)^{-i\omega} \right] \quad . \quad (4.4)$$

By using the formula

$$\lim_{a \rightarrow 0} \frac{\sin(x \ln a)}{x} = -\frac{\pi}{2} \delta(x) \quad , \quad (4.5)$$

where the delta function is normalized on the half axis, one can define the limiting value of $K_{i\omega}(\mu\rho)$ as the distribution

$$\lim_{\rho \rightarrow 0} K_{i\omega}(\mu\rho) = \frac{\pi}{2} \delta(\omega) \quad . \quad (4.6)$$

The consequence of (4.2) and (4.6) is that in the presence of the non-minimal coupling classical energy and Hamiltonian are affected by the soft modes in the different way. Consider, for example, a wave packet $\phi_{\Delta\omega}(t, \rho, z)$ which is constructed of soft modes with frequencies ω in the range $(0, \Delta\omega)$. Let function $\phi_{\Delta\omega}(t, \rho, z)$ be a solution of the Klein-Gordon equation of the form

$$\phi_{\Delta\omega}(t, \rho, z) = \frac{1}{2\pi} \int_0^\infty d\omega \int d^2k u_{\omega,k}(t, \rho) e^{-ik_j z^j} \tilde{\phi}_{\Delta\omega}(\omega) \tilde{\varphi}(k) \quad . \quad (4.7)$$

Here $\tilde{\phi}_{\Delta\omega}(\omega)$ and $\tilde{\varphi}(k)$ are some functions of ω and k_j , and

$$\tilde{\phi}_{\Delta\omega}(\omega) = 0 \quad , \quad \text{if } \omega > \Delta\omega \quad . \quad (4.8)$$

To have a non-zero value $\phi_{\Delta\omega}(t, \rho, z)$ at $\rho \rightarrow 0$ we assume that

$$\tilde{\phi}_{\Delta\omega}(\omega) \simeq \frac{2}{\sqrt{\pi\omega}} \quad , \quad \text{when } \omega \rightarrow 0 \quad . \quad (4.9)$$

Then

$$\lim_{\rho \rightarrow 0} \phi_{\Delta\omega}(t, \rho, z) = \varphi(z) \quad , \quad (4.10)$$

where $\varphi(z)$ is defined as

$$\varphi(z) = \frac{1}{2\pi} \int d^2k e^{-ik_i z^i} \tilde{\varphi}(k) \quad .$$

The canonical energy of this wave packet, given by the integral

$$H[\phi_{\Delta\omega}] = \int_0^\infty d\omega \omega |\tilde{\phi}_{\Delta\omega}(\omega)|^2 \int d^2z |\varphi(z)|^2 \quad , \quad (4.11)$$

can be made arbitrary small as $\Delta\omega \rightarrow 0$. On the other hand, the value of $\phi_{\Delta\omega}$ on the horizon does not vanish so that the energy E of the wave packet

$$E[\phi_{\Delta\omega}] = -2\pi\xi\beta_H^{-1} \int d^2z |\varphi(z)|^2 \quad (4.12)$$

remains non-zero.

We demonstrate now how the soft modes generate the Noether charge \bar{Q} in quantum theory. Consider the quantum scalar field and calculate its correlator on the bifurcation surface Σ ($\rho = 0$). The correlator for the canonical ensemble of the Rindler particles at the temperature β^{-1} is defined as

$$G_\beta(x, x') = \langle \hat{\phi}(x) \hat{\phi}(x') \rangle_\beta = \text{Tr} [\hat{\rho}(\beta) \hat{\phi}(x) \hat{\phi}(x')] \quad , \quad (4.13)$$

where $\hat{\rho}(\beta)$ is the density matrix (2.11) (where β_H is replaced by β). Expression (4.13) can be rewritten as

$$\langle \hat{\phi}(x) \hat{\phi}(x') \rangle_\beta = \int_0^\infty d\omega \int d^2k \left[n_\omega(\beta) U_{\omega,k}^*(x) U_{\omega,k}(x') + (n_\omega(\beta) + 1) U_{\omega,k}(x) U_{\omega,k}^*(x') \right] \quad , \quad (4.14)$$

where $n_\omega(\beta)$ is the density of particles with the energy ω

$$n_\omega(\beta) = \left(e^{\kappa\beta\omega} - 1\right)^{-1} . \quad (4.15)$$

Eq.(4.14) follows from the decomposition of field operators in the Rindler basis

$$\hat{\phi}(x) = \int_0^\infty d\omega \int d^2k \left[U_{\omega,k}^*(x) \hat{b}^+(\omega, k) + U_{\omega,k}(x) \hat{b}(\omega, k) \right] , \quad (4.16)$$

and formula

$$\langle \hat{b}^+(\omega, k) \hat{b}(\omega', k') \rangle_\beta = \delta(\omega - \omega') \delta^{(2)}(k - k') n_\omega(\beta) , \quad (4.17)$$

where $\hat{b}^+(\omega, k)$ and $\hat{b}(\omega, k)$ are creation and annihilation operators of the Rindler particles. The restriction of the correlator $G_\beta(x, x')$ on the bifurcation surface Σ is obtained in the limit when coordinates ρ and ρ' of its both points x and x' tend to zero. According to Eq.(4.6), the main contribution to the correlator in this limit is given by the modes with negligibly small frequencies ω . The density number of such modes is singular and is approximated by the expression

$$n_\omega(\beta) \simeq \frac{1}{\kappa\beta\omega} .$$

So one finds for small ρ and ρ'

$$G_\beta(x, x') \simeq \frac{1}{2\pi^4 \kappa \beta} \int d^2k e^{ik(z-z')} \int_0^\infty d\omega \frac{\sinh \pi\omega}{\omega} K_{i\omega}(\mu\rho) K_{i\omega}(\mu\rho') . \quad (4.18)$$

The integration over ω can be done by making use of asymptotic (4.4) and property (4.5) and the result reads

$$\int_0^\infty d\omega \frac{\sinh \pi\omega}{\omega} K_{i\omega}(\mu\rho) K_{i\omega}(\mu\rho') \simeq -\frac{\pi^2}{4} \ln(\mu^2 \epsilon^2) . \quad (4.19)$$

Here ϵ is a constant with the dimensionality of a length, which is introduced to keep the expression in the logarithm dimensionless. In derivation of (4.19) we omitted terms which do not depend on μ . These terms give a contribution to $G_\beta(x, x')$ on Σ proportional to $\delta^{(2)}(z - z')$ and vanishing when $z \neq z'$. Denote by $\mathcal{G}_\beta(z, z')$ the limiting value of $G_\beta(x, x')$ on the bifurcation surface, where z and z' are the coordinates of the points x and x' on Σ . Because $\mu^2 = m^2 + k_j^2$ one finds from (4.18) the following expression

$$\mathcal{G}_\beta(z, z') = -\frac{1}{2(2\pi)^2 \kappa \beta} \int d^2k e^{ik_j(z-z')^j} \ln[(m^2 + k_j^2) \epsilon^2] . \quad (4.20)$$

When the arguments z and z' coincide the integral over k in expression (4.20) has to be regularized. This is a standard problem when one is dealing with the coincidence limit of Green functions. By assuming that such a regularization is carried out we find that the quantity $\mathcal{G}_\beta(z, z)$ determines a non-zero canonical average of the charge \hat{Q}

$$\langle \hat{Q} \rangle_\beta = 2\pi\xi \int_\Sigma \langle \hat{\phi}^2(z) \rangle_\beta d^2z = 2\pi\xi \int_\Sigma \mathcal{G}_\beta(z, z) d^2z . \quad (4.21)$$

The results of our analysis can be summarized in the following way. In classical theory the quantity Q is determined by the Rindler modes with zero frequencies. This property also holds in the quantum theory where the average $\langle \hat{Q} \rangle_\beta$ is not zero because the density $n_\omega(\beta)$ of the low-frequency modes is singular⁵. In fact one can conclude that only soft modes with $\omega \ll 1$ are responsible for the non-zero value of correlator (4.20).

5 Spectral density of states of a black hole in induced gravity

We return now to the problem of the statistical-mechanical origin of the black hole entropy in the induced gravity. Since the entropy of a black hole of mass M is $S^{BH} = 4\pi M^2/G$, one might expect that the density of states of such a black hole is [31],[32]

$$\nu_{BH}(M)\Delta M \sim \exp\left(\frac{4\pi M^2}{G}\right)\Delta M. \quad (5.1)$$

The statistical-mechanical foundation of black hole thermodynamics implies the explanation of this degeneracy. Let us discuss how this problem can be solved in the induced gravity. We shall see that soft modes introduced in the previous Section play an important role in this discussion.

In the "constituent representation" of the gravitational action we are dealing with the ultraheavy particles propagating in the given external background. The average energy of these particles is

$$\bar{E} = \langle E \rangle = \int \langle T_{\mu\nu} \rangle \zeta^\mu d\sigma^\nu = 0. \quad (5.2)$$

Hence the mass M_{BH} of the black hole measured at the horizon and the mass M_∞ measured at infinity are the same. This equality is the result of averaging over the states of the constituent fields. A particular mode of a quantum field contributes to the energy, and hence to the mass. This gives rise to the difference

$$M_\infty - M_{BH} = \Delta M = \Delta E = \int T_{\mu\nu} \zeta^\mu d\sigma^\nu \quad (5.3)$$

determined by the differential mass formula [33]. If one fixes the mass of the system measured at infinity the mass of the black hole is not fixed but fluctuates near its average value. The origin of these fluctuations (black hole mass "zitterbewegung") are the fluctuations of the quantum constituent fields in vicinity of the black hole. Moreover in the induced gravity the degeneracy (5.1) of the black hole can be related to the number $\nu(E)$ of physically different states of constituents propagating in the black hole exterior and having the total energy E in the interval $(0, \Delta M)$. Namely, we have

$$\nu_{BH}(M) = \nu(E = 0). \quad (5.4)$$

⁵Note that this is a specific property of the boson fields and this is not true for the fields with Fermi statistics.

The problem of counting the states of constituents is technically much simpler. We suggest that "physical" states together with soft mode states form a complete set that can be identified with the space of Rindler modes. For this reason in order to obtain $\nu(E)$ it is sufficient to find the corresponding spectral densities of states for the Rindler and soft modes. Strictly speaking to calculate the density of states one needs to know the properties of the system at the temperature different from the Hawking value, i.e. one must consider the so called "off-shell" configurations. However the "off-shellness" can be arbitrary small, and we shall see that only the properties of the states near the Hartle-Hawking equilibrium are really important.

6 Spectral density of Rindler states

We begin with analysis of the properties of the thermal canonical ensembles of Rindler particles⁶. Let $Z_R(\beta)$ be the statistical-mechanical partition function of the complete set of scalar and spinor constituents of the induced gravity model propagating in the Rindler-like wedge

$$Z_R(\beta) = \text{Tr } e^{-\beta \hat{H}} = \prod_i \text{Tr}_i e^{-\beta \hat{H}_i} \quad . \quad (6.1)$$

Here $\hat{H} = \sum \hat{H}_i$ is the total Hamiltonian, and \hat{H}_i are the Hamilton operators for the each particular constituent. For the Hartle-Hawking vacuum $\beta = \beta_H$. We shall use subscript R to refer to quantities that are obtained by means of $Z_R(\beta)$.

The energy E_R and entropy S_R of the Rindler modes are defined as

$$E_R = -\frac{\partial}{\partial \beta} \ln Z_R(\beta)|_{\beta=\beta_H} \quad , \quad (6.2)$$

$$S_R = -\left(\beta \frac{\partial}{\partial \beta} - 1\right) \ln Z_R(\beta)|_{\beta=\beta_H} \quad . \quad (6.3)$$

Another representation for these quantities is

$$E_R = \langle \hat{H} \rangle = \sum_i \langle \hat{H}_i \rangle \equiv \sum_i \text{Tr}_i (\hat{\rho}_i \hat{H}_i) \quad , \quad (6.4)$$

$$S_R = -\sum_i \text{Tr}_i \hat{\rho}_i \ln \hat{\rho}_i \quad . \quad (6.5)$$

In (6.4) we made use of the thermalization theorem [30] according to that the statistical-mechanical average at $\beta = \beta_H$ of the operators localized in the wedge is equivalent to the quantum-mechanical average in the Hartle-Hawking vacuum.

The Rindler partition function $Z_R(\beta)$ can be found explicitly [16] and it is given by Eqs.(2.12) and (2.13). It can be presented as

$$Z_R(\beta) = \exp(-\beta \mu_R + \beta^{-1} \lambda_R) \quad . \quad (6.6)$$

⁶Since we are dealing with a black hole the term "Boulware" might be more appropriate. We use a term "Rindler" both following tradition and in order to stress the nature of our approximation.

By making use of Eqs.(2.12) and (2.13) one finds that

$$\lambda_R = \frac{\beta_H}{2} S_R = \frac{1}{3} g (4\pi M)^3 \quad , \quad (6.7)$$

$$E_R - \mu_R = \frac{1}{2\beta_H} S_R = \frac{\pi}{3} g M \quad , \quad (6.8)$$

where $g = \sum_i g(m_i^2)$.

The spectral density $\nu_R(E)$ of the operator \hat{H}_R is defined in the standard way

$$\nu_R(E) = \text{Tr}_R \delta(\hat{H} - E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\alpha \text{Tr}_R e^{i\alpha(E-\hat{H})} = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\alpha e^{i\alpha E} Z_R(i\alpha) \quad . \quad (6.9)$$

The integration in (6.9) can be performed [34]. For $E \geq \mu_R$ one finds

$$\nu_R(E) = \delta(E - \mu_R) + \tilde{\nu}_R(E) \quad , \quad (6.10)$$

$$\tilde{\nu}_R(E) = \left(\frac{\lambda_R}{E - \mu_R} \right)^{1/2} I_1 \left(2\sqrt{\lambda_R(E - \mu_R)} \right) \quad , \quad (6.11)$$

where $I_1(x)$ is the modified Bessel function. The function $Z_R(\beta)$ is obtained from $\nu_R(E)$ with the help of the Laplace transformation

$$Z_R(\beta) = e^{-\beta\mu_R} + \int_{\mu_R}^{\infty} e^{-\beta E} \tilde{\nu}_R(E) dE \quad . \quad (6.12)$$

It is possible to show that the spectrum of \hat{H}_R is positive ($\mu_R > 0$).

The function $w_R(E, \beta)$ describing the probability density to find the energy of the corresponding canonical ensemble at the temperature β^{-1} in the energy interval between E and $E + dE$ is

$$w_R(E, \beta) = Z_R(\beta)^{-1} \nu_R(E) e^{-\beta E} \quad . \quad (6.13)$$

According to (6.9), it is normalized to unity.

We shall be interested in the probability density at the Hawking temperature $w_R(E) \equiv w_R(E, \beta_H)$. Since $E_R - \mu_R \gg \lambda_R^{-1} > 0$ and we are interested in the energy region near E_R , we can use the asymptotics of the Bessel functions to obtain from (6.11)

$$w_R(E) \simeq \frac{1}{(4\pi)^{1/2} Z_R(\beta_H)} \left(\frac{\lambda_R}{(E - \mu_R)^3} \right)^{1/4} e^{f_R(E) - \beta_H E} \quad , \quad (6.14)$$

$$f_R(E) = 2\sqrt{\lambda_R(E - \mu_R)} \quad . \quad (6.15)$$

Then it is easy to show that function (6.14) can be approximated by a Gauss distribution with the center at $E = E_R$. One can check with the help of Eq.(6.6) that near the maximum

$$f_R(E) - \beta_H E \simeq \ln Z_R(\beta_H) - \frac{(E - E_R)^2}{\sigma_R^2} \quad , \quad (6.16)$$

where $\sigma_R^2 = 4\lambda_R/\beta_H^3 = g/24$. So that we have

$$w_R(E) \simeq \frac{1}{\sigma_R \sqrt{\pi}} e^{-\frac{(E-E_R)^2}{\sigma_R^2}} . \quad (6.17)$$

The parameter σ_R gives the width of the peak and it is of the order of magnitude of the mass-parameter, characterizing the cut-off scale. It depends on the regularization scheme. For instance, in the Pauli-Villars regularization [21] σ_R is the largest mass of the auxiliary Pauli-Villars fields. The appearance of the ultraviolet cut-off in distribution (6.14) is explained by the fact that the function $Z_R(\beta)$ has the ultraviolet divergencies which have to be regularized.

One can easily obtain the Rindler density of states $\nu_R(E)$ at the peak of the probability distribution. Substituting (6.14) into (6.13) one has

$$\nu_R(E_R) \sim \exp f_R(E_R) = \exp S_R , \quad (6.18)$$

where $S_R = \frac{\pi}{3}g\mathcal{A}^H$ and $\mathcal{A}^H = 16\pi M^2$ is the surface area of the black hole. As expected the logarithm of $\nu_R(E_R)$ is the entropy of the canonical ensemble of the Rindler particles.

7 Probability distribution and degeneracy of black hole states

Let us establish now the relation between degrees of freedom of Rindler particles and excitation states of a black hole. First of all we note that by using Eqs. (2.10) and (3.11) one can relate the Rindler energy and entropy to the total average energy \bar{E} and Bekenstein-Hawking entropy S^{BH}

$$S_R = S^{BH} + \bar{Q} , \quad (7.1)$$

$$E_R = \bar{E} + \beta_H^{-1}\bar{Q} . \quad (7.2)$$

Here

$$\bar{Q} = \langle \hat{Q} \rangle = 2\pi \sum_s \xi_s g(m_s^2) \mathcal{A}^H . \quad (7.3)$$

Since in the induced gravity on Ricci-flat backgrounds $\bar{E} = 0$ one has

$$E_R = \beta_H^{-1}\bar{Q} . \quad (7.4)$$

Substituting this relation into (6.8) one defines the parameter μ_R in the Rindler partition function (6.6).

As we mentioned, at least some of the non-minimal coupling constants ξ_s must be positive. In what follows we assume for simplicity that all $\xi_s > 0$, and hence the charge \bar{Q} is positive. The generalization to the case where this assumption is not satisfied is straightforward.

Relations (7.1) and (7.2) indicate that only part of the total energy and entropy of Rindler modes is responsible for thermal characteristics of a black hole. It suggests that the total system described by Rindler modes consists in fact of two independent parts, one is connected with "physical" degrees of freedom of the black hole, and the other is a subsystem of soft modes. The soft modes do not contribute to the canonical energy. In other words, one can add any number of soft modes to the given state without changing its Rindler energy. Therefore we identify the space of "physical" states with the space of Rindler states modulo the subspace of soft modes. We demonstrate that this identification correctly reproduces the degeneracy of the black hole mass.

In accordance with relations (7.1) and (7.2) and our assumption the partition function obeys the factorization property

$$Z_R(\beta) = Z(\beta) Z_Q(\beta) \quad . \quad (7.5)$$

Here $Z(\beta)$ and $Z_Q(\beta)$ are partition functions for "physical" and soft modes, respectively. Equations (7.1) and (7.2) follow from (7.5) provided

$$\beta_H^{-1} \bar{Q} = -\frac{\partial}{\partial \beta} \ln Z_Q(\beta)|_{\beta=\beta_H} \quad , \quad (7.6)$$

$$\bar{Q} = -\left(\beta \frac{\partial}{\partial \beta} - 1\right) \ln Z_Q(\beta)|_{\beta=\beta_H} \quad . \quad (7.7)$$

We recall that \bar{Q} and Z_R are divergent and so some regularization in (7.5) is supposed. It is important that all three partition functions that enter this relation are regularized by using the same regularization scheme. Eqs. (7.6) and (7.7) imply that

$$Z_Q(\beta_H) = 1 \quad . \quad (7.8)$$

One can introduce the density of states $\nu(E)$ for $Z(\beta)$ as

$$\nu(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\alpha e^{i\alpha E} Z(i\alpha) \quad , \quad (7.9)$$

so that

$$Z(\beta) = \int_{\mu}^{\infty} dE e^{-\beta E} \nu(E) \quad . \quad (7.10)$$

Similar relations are used to define $\nu_Q(E)$ in terms of $Z_Q(\beta)$. We assume that $\nu(E)$ and $\nu_Q(E)$ are non-vanishing only for $E \geq \mu$ and $E \geq \mu_Q$, respectively, where μ and μ_Q are some constants. Their value will be specified later. The factorization property (7.5) implies the following relation between the densities of states

$$\nu_R(E) = \int_{\mu}^{E-\mu_Q} \nu(E') \nu_Q(E-E') dE' \quad . \quad (7.11)$$

We can also define by the relations similar to (6.13) the probability distributions $w(E)$ and $w_Q(E)$ for each of the subsystems at $\beta = \beta_H$. Then equation (7.11) and factorization formula (7.5) result in the following relation

$$w_R(E) = \int_{\mu}^{E-\mu_Q} dE' w(E') w_Q(E-E') \quad . \quad (7.12)$$

Important properties of the distribution w_Q for the soft modes are determined by Eqs. (7.6) and (7.7). Namely let us write

$$\nu_Q(E) = \exp f_Q(E) \quad , \quad (7.13)$$

and assume that the function $f_Q(E)$ grows at infinity slower than E . In this case the probability distribution $w_Q(E)$ has a maximum at the point E_Q where $f'_Q(E_Q) = \beta_H$. Near this maximum

$$w_Q(E) \sim \frac{1}{\sigma_Q \sqrt{\pi}} \exp \left[-\frac{(E - E_Q)^2}{\sigma_Q^2} \right] \quad , \quad (7.14)$$

where $\sigma_Q^{-2} = \frac{1}{2} |f''_Q(E_Q)|$. Note that Eq.(7.6) can be rewritten as

$$\beta_H^{-1} \bar{Q} = \int_{\mu_Q}^{\infty} dE \ E \ w_Q(E) \quad . \quad (7.15)$$

So by using the Gaussian approximation (7.14) we get

$$E_Q \simeq \beta_H^{-1} \bar{Q} = E_R \quad (7.16)$$

and E_Q turns out to be the average energy for the canonical ensemble of the soft modes. On the other hand, Eq.(7.8) gives

$$f_Q(E_Q) \simeq \beta_H E_Q \quad . \quad (7.17)$$

Consequently the density number of states $\nu_Q(E)$ at the peak $E = E_Q$ is

$$\nu_Q(E) \simeq \exp(\beta_H E_Q) \simeq \exp \bar{Q} \quad . \quad (7.18)$$

According to the last equation the Noether charge \bar{Q} can be interpreted as the entropy of the soft modes.

In the Gaussian approximation (7.14) equation (7.12) takes the form

$$\frac{1}{\sigma_R \sqrt{\pi}} e^{-\frac{(E-E_R)^2}{\sigma_R^2}} = \int dE' \ w(E') \ \frac{1}{\sigma_Q \sqrt{\pi}} e^{-\frac{(E-E'-E_Q)^2}{\sigma_Q^2}} \quad , \quad (7.19)$$

where $E_R \simeq E_Q$. It follows from (7.19) that the probability distribution $w(E)$ also has the Gaussian form

$$w(E) \sim \frac{1}{\sigma \sqrt{\pi}} e^{-\frac{E^2}{\sigma^2}} \quad , \quad (7.20)$$

with the dispersion σ

$$\sigma^2 = \sigma_R^2 - \sigma_Q^2 \quad . \quad (7.21)$$

Now by taking into account Eq.(7.8) and the fact that the distribution of the soft modes has the peak at $E = E_Q$ we find from (7.11)

$$\nu_R(E) \sim \nu(E - E_Q) \nu_Q(E_Q) \quad . \quad (7.22)$$

The distribution (7.20) of the "physical" modes is centered at $E = 0$. The density of states at the maximum of the function $w(E)$, according to (7.22), is

$$\nu(0) \sim \frac{\nu_R(E_R)}{\nu_Q(E_Q)} \sim \exp(S_R - \bar{Q}) \quad . \quad (7.23)$$

By using relation (7.1) one can rewrite this expression as

$$\nu(0) \sim \exp S^{BH} = \exp \frac{\mathcal{A}^H}{4G} \quad . \quad (7.24)$$

Thus we proved that the spectrum of the "physical" states correctly reproduces the degeneracy of black hole mass levels. Therefore the identification of the black hole states with the "physical" states is justified. As the result the distribution of the black hole mass induced by quantum fluctuations of the constituents is centered near the average value M and has the width σ .

The following arguments can be used now to obtain an additional information concerning the width σ . By assuming that the function $Z_Q(\beta)$ has the temperature asymptotic similar to that of $Z_R(\beta)$, see Eq.(6.6), one can write

$$Z_Q(\beta) = \exp(-\beta\mu_Q + \beta^{-1}\lambda_Q) \quad . \quad (7.25)$$

The parameters μ_Q and λ_Q can be found from Eqs.(7.6) and (7.7)

$$\lambda_Q = \frac{1}{2}\beta_H\bar{Q}, \quad \mu_Q = \frac{1}{2\beta_H}\bar{Q} \quad . \quad (7.26)$$

Using these relations we get

$$Z_Q(\beta) = \exp \left[-\frac{1}{2} \left(\frac{\beta}{\beta_H} - \frac{\beta_H}{\beta} \right) \bar{Q} \right] \quad . \quad (7.27)$$

For this partition function $\sigma_Q^2 = 4\lambda_Q\beta_H^{-3} = \frac{1}{4}\sum_s \xi_s g_s$. Hence, according to (7.21) and (2.18),

$$\sigma^2 = \frac{1}{24} \sum_i g(m_i^2) - \frac{1}{4} \sum_s \xi_s g(m_s^2) = \frac{1}{32\pi G} \quad . \quad (7.28)$$

In other words, the width σ of the probability distribution of the black hole states does not depend on the regularization ambiguity, it is finite and proportional to the Planck mass $m_{Pl} = G^{-1/2}$.

The factorization property (7.5) together with (7.27) implies that $Z(\beta)$ has the same form as $Z_R(\beta)$ and $Z_Q(\beta)$ and can be written explicitly as

$$Z(\beta) = \exp \left[\frac{1}{2} \left(\frac{\beta}{\beta_H} + \frac{\beta_H}{\beta} \right) S^{BH} \right] \quad . \quad (7.29)$$

The important property of the partition function of the "physical" degrees of freedom in the induced gravity is that it is defined entirely by the ultraviolet finite quantity S^{BH} (at least in the one-loop approximation). So the function $Z(\beta)$, even if it is taken off-shell, i.e. for an arbitrary temperature β^{-1} , is well defined and ultraviolet finite.

8 Soft modes and fluctons

The degrees of freedom which enable one to single out "physical" states from the space of Rindler states are associated with the soft modes. Yet an explicit formulation of the black hole statistical-mechanics in terms of "physical" degrees of freedom is a non-trivial problem. To some extent this reminds the situation in the gauge theories where in general the constraints cannot be resolved explicitly. In many cases, however, it is sufficient to describe the physical space indirectly as a factorization of an extended space over the group of gauge transformations. In the functional integral such a factorization is realized by introducing the Faddeev-Popov ghosts.

Let us make some additional remarks concerning the subsystem of the soft modes. We saw that these modes are located in the nearest vicinity of the horizon, thus their physics is two dimensional by its nature. Moreover we will demonstrate now that the charge \bar{Q} can be expressed as an effective action of a two-dimensional quantum theory.

To this aim we use representation (4.20) for the correlator of the scalar field on the bifurcation surface Σ . In the Hartle-Hawking state ($\beta = \beta_H$) formula (4.20) reads

$$\mathcal{G}(z, z') = -\frac{1}{4\pi} \langle z | \ln((-\nabla_\Sigma^2 + m^2)\epsilon^2) | z' \rangle \quad , \quad (8.1)$$

where $-\nabla_\Sigma^2$ is the Laplace operator on Σ . Thus one can write

$$2\pi \int_\Sigma \langle \hat{\phi}(z)^2 \rangle d^2z = 2\pi \int_\Sigma \mathcal{G}(z, z) d^2z = -\frac{1}{2} \ln \det((-\nabla_\Sigma^2 + m^2)\epsilon^2) \quad . \quad (8.2)$$

The quantity $W_\chi = \frac{1}{2} \ln \det[(-\nabla_\Sigma^2 + m^2)\epsilon^2]$ is identical to the effective action for a two-dimensional quantum field χ defined on Σ . The functional W_χ can be rewritten as the Euclidean functional integral over the field χ

$$e^{-W_\chi} = \int D[\chi] \exp \left[-\frac{1}{2} \int_\Sigma ((\nabla_\Sigma \chi)^2 + m^2 \chi^2) d^2z \right] \quad . \quad (8.3)$$

We call χ *flucton* field to distinguish it from the original scalar field ϕ . The flucton field is the free field with the same mass m as the 4D field ϕ . According to Eq.(8.1), the quantum theory of fluctons is completely defined by the 4D correlator $\mathcal{G}(z, z')$ on Σ .

From (8.3) one obtains the representation for the charge \bar{Q}

$$\bar{Q} = 2\pi\xi \int_\Sigma \langle \hat{\phi}(z)^2 \rangle d^2z = -\xi W_\chi \quad . \quad (8.4)$$

As was shown in Section 7, the quantity \bar{Q} coincides with the entropy of the ensemble of the soft modes (see Eq. (7.18)). On the other hand, the integral (8.3) can be interpreted as microcanonical partition function of the flucton fields and W_χ as the microcanonical free energy. Consequently, $W_\chi = -S_\chi$, where S_χ is the entropy of fluctons. These arguments enable one to represent the Noether charge in a pure statistical mechanical form

$$\bar{Q} = -\xi W_\chi = \xi S_\chi \quad (8.5)$$

and relate it to the entropy of a microcanonical ensemble of two-dimensional fields χ on Σ . The constant of the non-minimal coupling ξ plays in (8.5) a role of the effective number of the flucton fields.

To derive Eq. (8.5) we used the Rindler approximation for the black hole geometry. It is possible to show (see Appendix) how to extend this two-dimensional interpretation to the case of arbitrary black hole backgrounds.

9 Discussion

We make now some general remarks concerning the derivation of the Bekenstein-Hawking entropy by counting the degrees of freedom of constituents in the induced gravity. First, let us compare the induced gravity with other theories. In general case one has for the observable value of the Newtonian constant G the following expression $G^{-1} = G_{bare}^{-1} + G_q^{-1}$, where G_q^{-1} is a (one-loop) quantum correction to the initial bare constant G_{bare}^{-1} . In order to obtain the finite value of G one usually begins with the infinite quantity G_{bare}^{-1} which absorbs the ultraviolet divergences. As the result in the expression for the black hole entropy besides the part $\mathcal{A}^H/(4G_q)$ that can be connected with statistical mechanics there is always the term $\mathcal{A}^H/(4G_{bare})$ having no clear statistical-mechanical meaning. In the induced gravity $G_{bare}^{-1} = 0$ and this problem is solved automatically.

In order to have the correct Einstein low energy gravity with the finite observable Newton constant one must impose special constraints on the parameters of the fields inducing the gravity. In our particular model these requirements are satisfied because of the presence of the non-minimally coupled fields. The same set of constraints guarantees that the induced entropy of the black hole is also finite and coincides with the Bekenstein-Hawking entropy S^{BH} . The entropy S^{BH} can be obtained from the statistical-mechanical entropy of the constituents by subtracting the Noether charge Q of the non-minimally coupled fields. The same quantity Q determines the difference between the energy E of the fields in the black hole exterior and the value of their Hamiltonian (canonical energy) H . We showed that there exist a set of states (soft modes) that contribute to Q but do not contribute to H , so that the Hamiltonian for the Rindler particles is degenerate. By using the factorization of the space of states of the Rindler particles with respect to the subspace of soft modes we obtained the degeneracy of black hole states responsible for the Bekenstein-Hawking entropy.

This mechanism is universal in the sense that it does not depend on the concrete choice of the set of scalar and spinor constituents and their properties provided the general constraints are satisfied. The concrete model of the induced gravity may differ from the one considered in this paper, and may contain, for example, finite or infinite number of fields of higher spins. However our consideration indicates that it is quite plausible that the same mechanism still works.

The obtained statistical-mechanical representation of black hole entropy in the induced

gravity does not depend on the particular structure of the theory at the Planck energies. All the information about field species and masses of the heavy constituents in the low-energy limit is compressed in the Newton constant (2.6). So any two microscopically different theories, that induce at low energies the same theory of gravity, predict the same entropy S^{BH} for a black hole. The details and the way of statistical-mechanical calculations of S^{BH} may depend on the type of the theory, but the results of calculations will coincide. The assumption that this happens in all theories having the Einstein gravity in the low energy limit was called in Ref.[16] the *low-energy censorship conjecture*.

It should be emphasized that we do not consider Sakharov's approach as a version of the final theory of quantum gravity. Certainly, it cannot compete with the superstring theory, which is considered as a modern candidate for quantum gravity theory. There are many indications that the superstring models give the correct answer for the black hole entropy by counting string degrees of freedom. But we would like to stress that the string calculations essentially use supersymmetry and usually deal with the black holes close to the extreme ones. Moreover for each model and type of a black hole the proof of the corresponding result requires new calculations and is often considered as a miracle (see e.g [37]).

Since the thermodynamical characteristics of macroscopical black holes are determined by a low-energy effective theory of gravity it is reasonable to suggest that there exists some mechanism that guarantees this universality. The models of induced gravity might be interesting as some kind of the phenomenological models in which many details concerning the underlying microscopical theory are lost, and only a few of its most important features are preserved. In particular in Sakharov's approach, as well as in the string theory, the gravity is the induced phenomenon and the Newton constant is ultraviolet finite. Our analysis indicates that namely these two features are sufficient for the statistical-mechanical explanation of the Bekenstein-Hawking entropy. That is why we believe that the proposed mechanism of black hole entropy generation may be of more general interest.

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A Noether charge of a non-minimally coupled scalar field as a 2D effective action

Let us consider the correlator of quantum scalar field ϕ , see (3.1), on the bifurcation surface Σ of the black hole horizons

$$\langle \hat{\phi}(x(z)) \hat{\phi}(x(z')) \rangle = \mathcal{G}(z, z') \quad . \quad (\text{A.1})$$

As earlier $\mathcal{G}(z, z')$ is the value of the Green $G(x, x')$ function with its both arguments taken on Σ . The arguments z and z' are the coordinates of the points x and x' on Σ , analogous to the coordinates z on the Rindler horizon in metric (4.1). Here we will be interested in function (A.1) on a general black hole background. We assume that the quantum state is the Hartle-Hawking vacuum. Then function $\mathcal{G}(z, z')$ has an Hadamard form and one can write for it the following Schwinger-DeWitt representation [35]

$$\mathcal{G}(z, z') = \Delta^{1/2}(z, z') \int_{\delta}^{\infty} ds \frac{1}{(4\pi s)^2} e^{-\frac{\sigma^2(z-z')}{4s} - m^2 s} \left(1 + a_1(z, z')s + a_2(z, z')s^2 + \dots \right) \quad . \quad (\text{A.2})$$

Here δ is ultraviolet cut-off parameter, $\sigma(z - z')$ is the 4D geodesic distance between the points, and

$$\Delta(x, x') = -[g(x)g(x')]^{-1/2} \det \left(\frac{1}{2} \frac{\partial^2 \sigma(x, x')}{\partial x^\mu \partial x^\nu} \right)$$

is the Van Vleck determinant. $a_i(x, x')$ are the coefficients of the asymptotic expansion of the heat kernel of the scalar wave operator $L = -\square + m^2 + \xi R$, see Eq. (3.1). The integration contour in (A.2) can be chosen real.

It is important to note that a two-dimensional geodesic on Σ is also a geodesic in the enveloping space-time. Indeed, because Σ is the fixed set of the Killing field, both second fundamental forms of Σ vanish. This is a necessary and sufficient condition for Σ to be a totally geodesic surface [36]. Thus one can substitute instead of $\sigma(z, z')$ in (A.2) the 2D geodesic distance $\sigma_\Sigma(z, z')$ on Σ . Let us consider now a two-dimensional operator

$$O_\Sigma = -\nabla_\Sigma^2 + m^2 + V[\mathcal{R}] \quad , \quad (\text{A.3})$$

where $-\nabla_\Sigma^2$ is the Laplacian on Σ , and $V[\mathcal{R}]$ is a "potential" which may depend on the scalar curvature R_Σ of Σ , and an external geometry in the vicinity of this surface. The matrix element of the heat kernel of O_Σ has the following asymptotic form

$$\langle z | e^{-s O_\Sigma} | z' \rangle \simeq \frac{\Delta_\Sigma^{1/2}(z, z')}{(4\pi s)} e^{-\frac{\sigma_\Sigma^2(z, z')}{4s} - m^2 s} \left(1 + a_{\Sigma,1}(z, z')s + a_{\Sigma,2}(z, z')s^2 + \dots \right) \quad , \quad (\text{A.4})$$

where $a_{\Sigma,i}(z, z')$ and $\Delta_\Sigma^{1/2}(z, z')$ are the heat kernel coefficients and the Van Vleck determinant on Σ , respectively.

Suppose now that the potential $V[\mathcal{R}]$ of the operator (A.3) on Σ can be chosen so that

$$a_i(z, z') \simeq a_{\Sigma,i}(z, z') \quad . \quad (\text{A.5})$$

Then by comparing (A.4) with (A.2) and taking into account that Σ is a totally geodesic surface one finds that

$$\mathcal{G}(z, z') \simeq \frac{1}{4\pi} e^{f(z, z')} \int_{\delta}^{\infty} \frac{ds}{s} \langle z | e^{-s O_{\Sigma}} | z' \rangle = -\frac{1}{4\pi} e^{f(z, z')} \langle z | \ln O_{\Sigma} | z' \rangle \quad . \quad (\text{A.6})$$

Here the logarithm of the operator is understood as a regularized quantity, in the same way as the correlator (A.2). The function $f(z, z')$ is defined as

$$e^{f(z, z')} = \frac{\Delta^{1/2}(z, z')}{\Delta_{\Sigma}^{1/2}(z, z')} \quad . \quad (\text{A.7})$$

For a spherical horizon the curvature decomposition of $f(z, z')$ looks as

$$f(z, z') = \frac{\sigma_{\Sigma}^2(z, z')}{24} (R - R_{\mu\nu} n_i^{\mu} n_i^{\nu} - R_{\Sigma}) + \dots \quad , \quad (\text{A.8})$$

where n_i^{μ} are two unit orthogonal vectors which are normal to Σ . The r.h.s. of (A.6) enables one an interpretation in terms of two-dimensional quantum theory of flucton field χ on Σ . Indeed, $f(z, z) = 0$ and

$$\frac{1}{2} \int_{\Sigma} \langle z | \ln O_{\Sigma} | z \rangle \sqrt{\gamma} d^2 z = \frac{1}{2} \ln \det(-\nabla_{\Sigma}^2 + m^2 + V[\mathcal{R}]) \equiv W_{\chi}[\gamma] \quad . \quad (\text{A.9})$$

The functional $W_{\chi}[\gamma]$ has the meaning of an effective action. It is expressed in terms the Euclidean path integral as

$$e^{-W_{\chi}[\gamma]} = \int D[\chi] \exp \left[-\frac{1}{2} \int_{\Sigma} (\chi^i \chi_{,i} + (m_s^2 + V[\mathcal{R}]) \chi^2) \sqrt{\gamma} d^2 z \right] \quad . \quad (\text{A.10})$$

where $D[\chi]$ is a covariant measure. From (A.9) one finds the representation of the Noether charge in terms of the effective action of flucton field χ

$$\bar{Q} = -\xi W_{\chi}[\gamma] \quad . \quad (\text{A.11})$$

Formula (A.11) is the generalization of Eq. (8.4) obtained in Section 8 in the Rindler approximation.

Let us note that in the case of the Rindler space $V[\mathcal{R}] = 0$, $f(z, z') = 0$ and basic equality (A.6) holds exactly. However, developing flucton theory on a general background is a more difficult problem. Such a theory may even not exist in a local form, if the relations (A.5) are not satisfied for $2D$ dimensional elliptic operators. However, this is not an obstruction for the models of the induced gravity based on the assumption that the Compton wave length λ of the heavy constituents is much smaller than the curvature radius of the background space. Because the field correlators vanish when $\sigma(z, z') \gg \lambda$, it is possible to satisfy Eq. (A.5) only approximately for the first coefficients and in some order in curvature. Moreover, to find the contribution to the Bekenstein-Hawking entropy (2.10) it is sufficient to calculate $\langle \hat{\phi}^2 \rangle$ by neglecting the curvature effects at all. That is why the Rindler approximation was justified in our analysis.

References

- [1] J.D. Bekenstein, Nuov. Cim. Lett. **4** (1972) 737.
- [2] J.D. Bekenstein, Phys. Rev. **D7** (1973) 2333; Phys. Rev. **D9** (1974) 3292.
- [3] S.W. Hawking, Comm. Math. Phys. **43** (1975) 199.
- [4] G.'t Hooft, Nucl. Phys. **B256** (1985) 727.
- [5] L. Bombelli, R. Koul, J. Lee, and R. Sorkin, Phys. Rev. **D34** (1986) 373.
- [6] M. Srednicki, Phys. Rev. Lett. **71** (1993) 666.
- [7] V. Frolov and I. Novikov, Phys. Rev. **D48** (1993) 4545.
- [8] V.P. Frolov, Phys. Rev. Lett. **74** (1995) 3319.
- [9] L. Susskind and J. Uglum, Phys. Rev. **D50** (1994) 2700.
- [10] C. Callan and F. Wilczek, Phys. Lett. **B333** (1995) 55.
- [11] A.A. Bytsenko, G. Cognola, and S. Zerbini, Nucl. Phys. **B458** (1996) 267.
- [12] A. Zelnikov and A. Barvinskii, *Density Matrix of a Black Hole and Bekenstein-Hawking Entropy*, to appear.
- [13] T. Jacobson, *Black Hole Entropy and Induced Gravity*, preprint gr-qc/9404039.
- [14] A.D. Sakharov, Sov. Phys. Doklady, **12** (1968) 1040, Theor. Math. Phys. **23** (1976) 435.
- [15] S.L. Adler, Rev. Mod. Phys. **54** (1982) 729.
- [16] V.P. Frolov, D.V. Fursaev, and A.I. Zelnikov, Nucl. Phys. **B486** (1997) 339.
- [17] R.M. Wald, Phys. Rev. **D48** (1993) R3427.
- [18] T.A. Jacobson, G. Kang, and R.C. Myers, Phys. Rev. **D49** (1994) 6587.
- [19] A.E. Mayo and J.D. Bekenstein, Phys. Rev. **D54** (1996) 5059.
- [20] S.N. Solodukhin, Phys. Rev. **D52** (1995) 7046.
- [21] J.-G. Demers, R. Lafrance, and R.C. Myers, Phys. Rev. **D52** (1995) 2245.
- [22] F. Larsen and F. Wilczek, Nucl. Phys. **B458** (1996) 249.

- [23] M. Hotta, T. Kato and K. Nagata, *A Comment on Geometric Entropy and Conical Space*, gr-qc/9611058, V. Moretti, *Geometric Entropy and Curvature Coupling in Conical Spaces: ζ Function Approach*, hep-th/9701099.
- [24] S.N. Solodukhin, *Non-Minimal Coupling and Quantum Entropy of a Black Hole*, hep-th/9612061.
- [25] C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation*, W.H. Freeman and Company, New York.
- [26] F. Belgiorno and S. Liberati, *Black Hole Thermodynamics, Casimir Effect and Induced Gravity*, gr-qc/9612024.
- [27] W. Nelson, Phys. Rev. **D50** (1994) 7400.
- [28] D.V. Fursaev and S.N. Solodukhin, Phys. Rev. **D52** (1995) 2143.
- [29] V. Iyer and R.M. Wald, Phys. Rev. **D52** (1995) 4430.
- [30] S. Takagi, Progress of Theoretical Physics Supplement **88** (1986).
- [31] S.W. Hawking, Phys. Rev. **D13** (1976) 191.
- [32] J.W. York, Phys. Rev. **D33** (1986) 2092.
- [33] J.M. Bardeen, B. Carter and S.W. Hawking, Commun. Math. Phys. **31** (1973) 161.
- [34] H. Bateman and A. Erdelyi, *Tables of Integral Transformations*, v.1, New York, McGraw-Hill Book Company, Inc., 1954.
- [35] B.S. DeWitt, *Dynamical Theory of Groups and Fields*, Gordon and Breach, New York 1965.
- [36] L.P. Eisenhart, *Riemannian Geometry*, Princeton University Press, 1966, Princeton.
- [37] G.T. Horowitz, *The Origin of Black Hole Entropy in String Theory*, gr-qc/9604051.